Verifiable Security of Boneh-Franklin Identity-Based Encryption

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Identity-Based Encryption (**IBE**)

Problem of standard **PKE**:

key management is involved and troublesome

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to use recipient's ID as public key

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Alice

Bob

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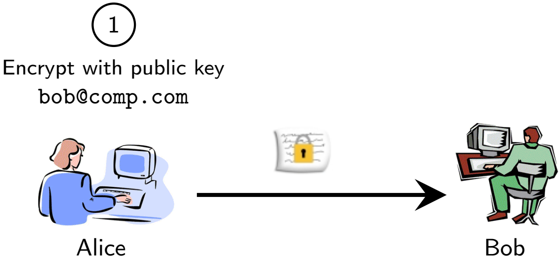

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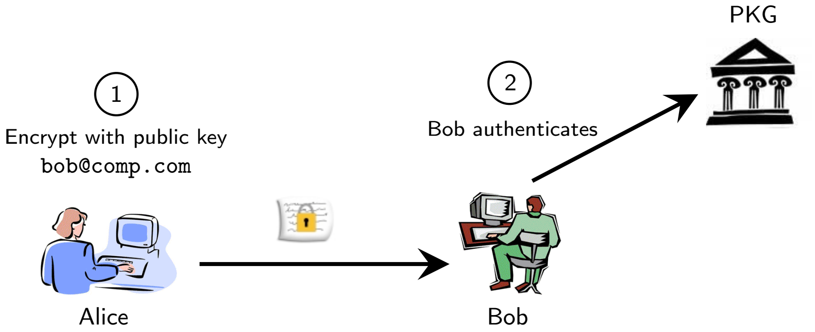

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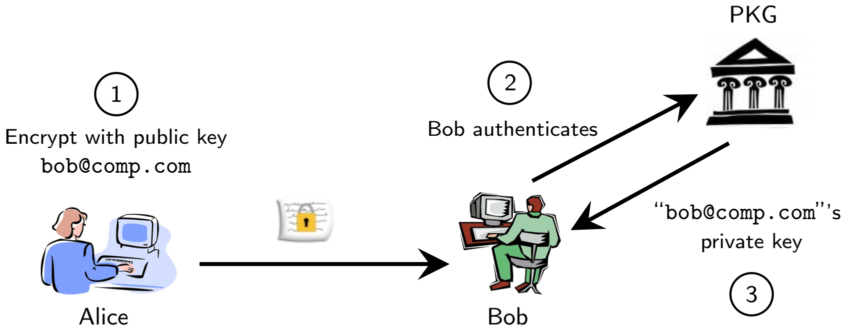
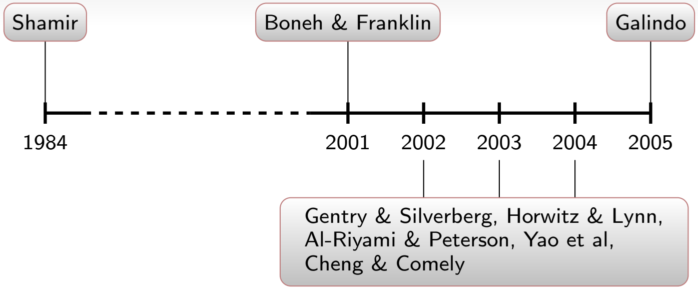


Should we rely on **IBE** schemes?



1984: Conception of identity-based cryptography

2001: First practical provably-secure **IBE** scheme.

2002-2005: Used as building block for many other protocols

2005: Security proof is flawed (but can be patched)

Improving the security argument

Verifiable security paradigm

Use formal methods to build certified security proofs of cryptographic systems

- Gives strong evidence of correctness of security arguments
- Enables *automation* in proofs
- Demonstrated *applicability* and *effectiveness*

Outline

1. The provably-secure `BasicIdent` scheme
2. CertiCrypt framework
3. Machine-checked proof of `BasicIdent` security
4. Summary and perspectives

An IBE Scheme

An *identity-based encryption scheme* is specified by four polynomial algorithms:

Setup

Encrypt

Extract

Decrypt

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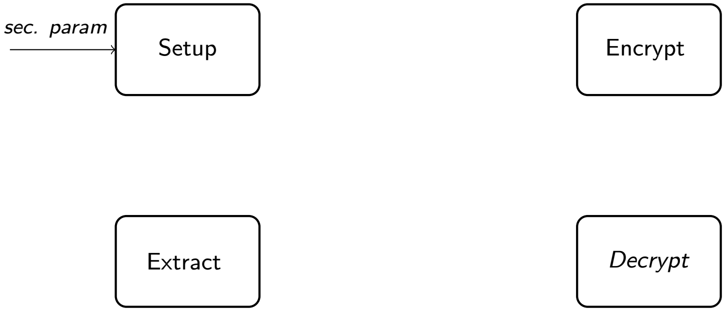

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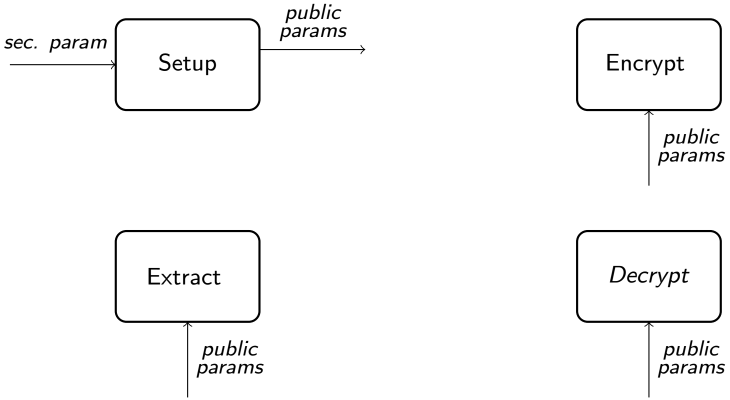

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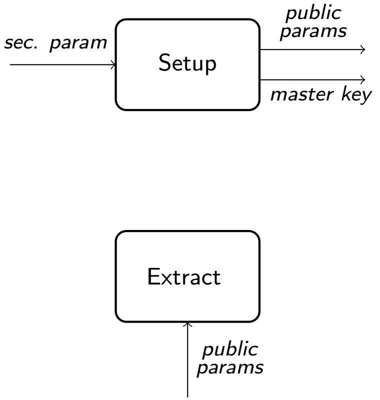

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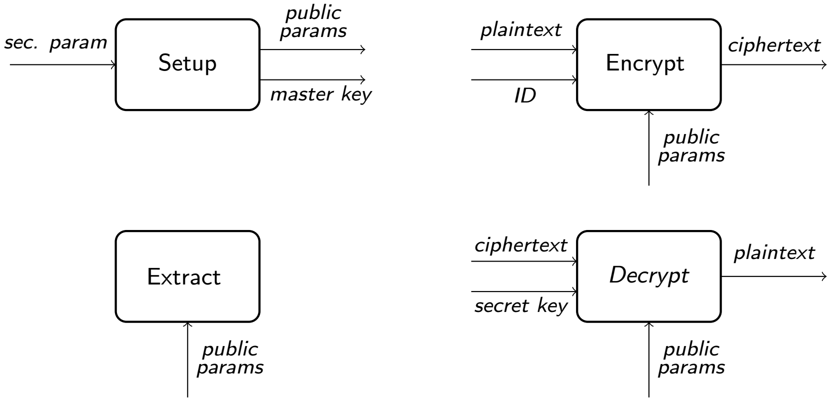

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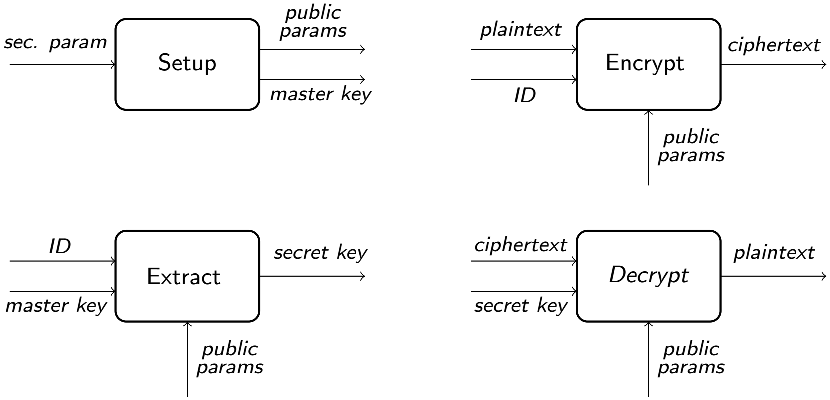


Boneh-Franklin's recipe

1. Extend the notions of IND-CPA and IND-CCA to **IBE** schemes
2. Build an IND-CPA-secure **IBE** scheme `BasicIdent`
3. Apply a variant of Fujisaki-Okamoto transformation to turn `BasicIdent` into an IND-CCA-secure **IBE** scheme

The BasicIdent scheme (definition)

Consider

- $\mathbb{G}_1$ and $\mathbb{G}_2$, two cyclic groups of prime order q,
- $\hat{e} : \mathbb{G}_1 \times \mathbb{G}_1 \to \mathbb{G}_2$, an efficiently computable bilinear map

$$\hat{e}(aP, bQ) = \hat{e}(P, Q)^{ab}$$
$$\langle P \rangle = \mathbb{G}_1 \implies \langle \hat{e}(P, P) \rangle = \mathbb{G}_2$$

- Two hash functions

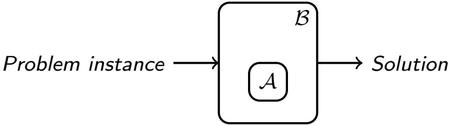
$$\mathcal{H}_1 : \{0,1\}^\star \to \mathbb{G}_1^+$$
$$\mathcal{H}_2 : \mathbb{G}_2 \to \{0,1\}^n$$

The `BasicIdent` **IBE**-scheme is defined as

Setup(k)	:	$P \stackrel{\$}{\leftarrow} \mathbb{G}_1^+;\ mk \stackrel{\$}{\leftarrow} \mathbb{Z}_q^+;\ P_{pub} \leftarrow mk \cdot P;$ return $((P, P_{pub}), mk)$
Extract(mk, ID)	:	$Q_{ID} \leftarrow \mathcal{H}_1(ID);$ return $mk \cdot Q_{ID}$
Encrypt(ID, m)	:	$Q_{ID} \leftarrow \mathcal{H}_1(ID);\ c \stackrel{\$}{\leftarrow} \mathbb{Z}_q^+;\ m' \leftarrow \mathcal{H}_2(e(Q_{ID}, P_{pub})^c);$ return $(c \cdot P, m \oplus m')$
Decrypt$(sk, (u, v))$	:	return $v \oplus \mathcal{H}_2(\hat{e}(sk, u))$

The `BasicIdent` scheme (security proof)

- Proof by reduction (in the random oracle model)
  - Define security goal (and adversarial model)
  - Consider a computational assumption
  - Reduce the security of the scheme to the intractability assumption.



$$\Pr\left[\begin{array}{l}\mathcal{A}\text{ breaks}\\ \text{the scheme}\end{array}\right] \leq \mathscr{F}\left(\Pr\left[\begin{array}{l}\mathcal{B}\text{ solves the}\\ \text{hard problem}\end{array}\right]\right)$$

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    ➡ **Indistinguishability under Chosen Plaintext Attack**
    Strengthened notion of ***PKE*** *IND-CPA for* ***IBE***
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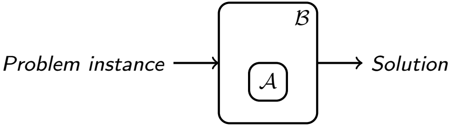

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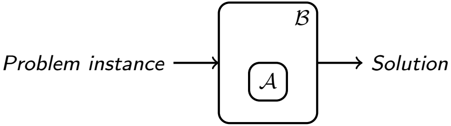
    ➥ **Bilinear Diffie-Hellman assumption**
    It is hard to compute $\hat{e}(P, P)^{abc}$ *given a random tuple* $(P, a \cdot P, b \cdot P, c \cdot P)$.
  - Reduce the security of the scheme to the intractability assumption.



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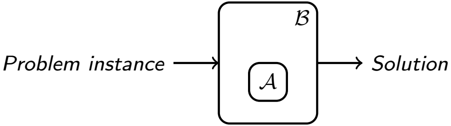

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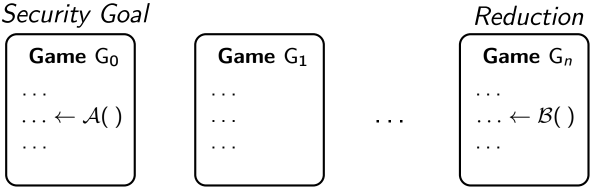
➥ $$\mathbf{Adv}^{\mathcal{A}}_{\text{IND-ID-CPA}} \leq \mathbf{Adv}^{\mathcal{B}}_{\text{BDH}} \; \frac{\exp(1)\; q_{\mathcal{H}_2}\; (1+q_{\mathcal{EX}})}{2}$$

Tidying the proof up

The game-playing technique



$$\Pr_{\mathsf{G_0}}[S_0] \;\le\; f_1\big(\Pr_{\mathsf{G_1}}[S_1]\big) \le \;\cdots\; \le f_n\big(\Pr_{\mathsf{G}_n}[S_n]\big)$$

CertiCrypt: machine-checked crypto proofs

Certified framework for building and verifying crypto proofs in the Coq proof assistant

- Combination of programming language techniques and cryptographic-specific tools
- Game-based methodology, natural to cryptographers
- Several case studies:
  - Encryption schemes: ElGamal, Hashed ElGamal, OAEP
  - Signature schemes: FDH, BLS
  - Zero-Knowledge protocols: Schnorr, Okamoto, Diffie-Hellman, Fiat-Shamir

Inside CertiCrypt (language syntax)

Language-based proofs

Formalize security definitions, assumptions and games using a probabilistic programming language.

pWhile: a probabilistic programming language

$$
\begin{array}{rcll}
\mathcal{C} & ::= & \text{skip} & \text{nop} \\
& \mid & \mathcal{C};\ \mathcal{C} & \text{sequence} \\
& \mid & \mathcal{V} \leftarrow \mathcal{E} & \text{assignment} \\
& \mid & \mathcal{V} \overset{\$}{\leftarrow} \mathcal{D} & \text{random sampling} \\
& \mid & \text{if } \mathcal{E} \text{ then } \mathcal{C} \text{ else } \mathcal{C} & \text{conditional} \\
& \mid & \text{while } \mathcal{E} \text{ do } \mathcal{C} & \text{while loop} \\
& \mid & \mathcal{V} \leftarrow \mathcal{P}(\mathcal{E}, \ldots, \mathcal{E}) & \text{procedure call}
\end{array}
$$

- $x \overset{\$}{\leftarrow} d$: sample the value of x according to distribution d
- The language of expressions ($\mathcal{E}$) and distribution expressions ($\mathcal{D}$) admits user-defined extensions

Observational equivalence

$$\models c_1 \simeq^I_O c_2$$

Example

$$\models x \xleftarrow{\$} \{0,1\}^k; y \leftarrow x \oplus z \simeq^{\{z\}}_{\{x,y,z\}} y \xleftarrow{\$} \{0,1\}^k; x \leftarrow y \oplus z$$

- Useful to relate probabilities

$$\frac{\mathrm{fv}(A) \subseteq O \quad \models c_1 \simeq^I_O c_2 \quad m_1 =_I m_2}{\Pr[c_1, m_1 : A] = \Pr[c_2, m_2 : A]}$$

Inside CertiCrypt (crypto-specific tool)

Fundamental lemma of game-playing

Game G_1	**Game** G_2
$\ldots$	$\ldots$
bad $\leftarrow$ true; c_1	**bad** $\leftarrow$ true; c_2
$\ldots$	$\ldots$

Two identical up to **bad** games

Lemma

If G_1 *and* G_2 *are identical up to* **bad**, *then*

$$|\Pr[G_1, m : A] - \Pr[G_2, m : A]| \leq \max\{\Pr[G_1, m : \mathbf{bad}], \Pr[G_2, m : \mathbf{bad}]\}$$

Our proof in CertiCrypt

We extended CertiCrypt with:

- Types and operators for the groups $\mathbb{G}_1, \mathbb{G}_2$
- An operator for a bilinear map $\hat{e} : \mathbb{G}_1 \times \mathbb{G}_1 \rightarrow \mathbb{G}_2$
- Simplification rules for computing normal forms of applications of the bilinear map $\hat{e}$
- An instruction for sampling from Bernoulli distributions

Formalizing the security goal:

Game $\mathsf{G}_{\mathsf{IND\text{-}ID\text{-}CPA}}$:
$(params, mk) \leftarrow \mathsf{Setup}(k);$
$(m_0, m_1, ID_{\mathcal{A}}) \leftarrow \mathcal{A}_1(params);$
$b \xleftarrow{\$} \{0,1\};$
$c \leftarrow \mathsf{Encrypt}(ID_{\mathcal{A}}, m_b);$
$b_{\mathcal{A}} \leftarrow \mathcal{A}_2(c)$

- The adversary is modeled by two procedures (of unknown code) $\mathcal{A}_1$ and $\mathcal{A}_2$ that communicate through shared variables
- $\mathcal{A}_1$ and $\mathcal{A}_2$ have oracle access to the extraction algorithm and to both random oracles
- Neither $\mathcal{A}_1$ nor $\mathcal{A}_2$ is allowed to query the challenge $ID_{\mathcal{A}}$ to the extraction oracle.

$$\mathbf{Adv}^{\mathcal{A}}_{\mathsf{IND\text{-}ID\text{-}CPA}} \stackrel{\text{def}}{=} \left| \Pr_{\mathsf{G}_{\mathsf{IND\text{-}ID\text{-}CPA}}} [b = b_{\mathcal{A}}] - \frac{1}{2} \right|$$

Formalizing the assumptions

- The Bilinear Diffie-Hellman assumption

Game $\mathsf{G}^{\mathcal{B}}_{\mathsf{BDH}}$:

$P \xleftarrow{\$} \mathbb{G}_1^+;\ a, b, c \xleftarrow{\$} \mathbb{Z}_q^+;$
$z \leftarrow \mathcal{B}(P, a \cdot P, b \cdot P, c \cdot P)$

$$\mathbf{Adv}^{\mathcal{B}}_{\mathsf{BDH}} \stackrel{\text{def}}{=} \Pr_{\mathsf{G}^{\mathcal{B}}_{\mathsf{BDH}}}\left[z = \hat{e}(P, P)^{abc}\right]$$

$$\forall \mathcal{B} \bullet \mathsf{PPT}(\mathcal{B}) \implies \mathsf{negl}(\mathbf{Adv}^{\mathcal{B}}_{\mathsf{BDH}})$$

- The random oracle model

Oracle $\mathcal{H}_1(ID)$:

if $ID \notin \mathsf{dom}(L_1)$ then
  $R \xleftarrow{\$} \mathbb{G}_1^+;$
  $L_1(ID) \leftarrow R$
return $L_1(ID)$

Oracle $\mathcal{H}_2(r)$:

if $r \notin \mathsf{dom}(L_2)$ then
  $m \xleftarrow{\$} \{0,1\}^n;$
  $L_2(r) \leftarrow m$
return $L_2(r)$

Our proof in CertiCrypt

Building the reduction...

Game $G_{\text{IND-ID-CPA}}$:

$(parm, mk) \leftarrow \text{Setup}(k);$
$(m_0, m_1, ID_{\mathcal{A}}) \leftarrow \mathcal{A}_1(parm);$
$b \xleftarrow{\$} \{0,1\};$
$c \leftarrow \text{Encrypt}(ID_{\mathcal{A}}, m_b);$
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...

Game $G^{\mathcal{B}}_{\text{BDH}}$:

$P \xleftarrow{\$} \mathbb{G}_1^+;\ a, b, c \xleftarrow{\$} \mathbb{Z}_q^+;$
$z \leftarrow \mathcal{B}(P, a\cdot P, b\cdot P, c\cdot P)$

$$\mathbf{Adv}^{\mathcal{A}}_{\text{IND-ID-CPA}} \leq \cdots \leq \mathbf{Adv}^{\mathcal{B}}_{\text{BDH}} \frac{\exp(1)\ q_{\mathcal{H}_2}\ (1+q_{\mathcal{EX}})}{2}$$

- Seven intermediate games
- Lazy sampling, fundamental lemma, Coron's technique
- Same bound as Boneh & Franklin proof

Our proof in CertiCrypt

- Our reduction is direct in contrast to Boneh-Franklin proof that goes through an intermediate IND-CPA-secure (non-IBE) encryption scheme
- Used a simpler argument instead of an inductive argument in Boneh-Franklin's proof that we could not reproduce
- 5000 lines of Coq script
- Built in 3 man-months (but automatically verifiable in 10 minutes)

Contributions

- Presented a machine-checked reduction of the security of the `BasicIdent` **IBE** scheme to the Bilinear Diffie-Hellman assumption
- Demonstrated that CertiCrypt can be extended to deal with complex security proofs of cryptographic schemes

Perspectives

- Formalize Fujisaki-Okamoto meta-result.
- Eliminate RO assumption on $\mathbb{G}_1$: formalize Brier *et al* work about indifferentiability of hash functions into elliptic curves.

Questions?

Get CertiCrypt (and EasyCrypt) from:
`http://certicrypt.gforge.inria.fr`

Inside CertiCrypt (language semantics)

Programs map an initial memory to a distribution of final memories:

$$[\![c \in \mathcal{C}]\!] : \mathcal{M} \to \mathcal{D}(\mathcal{M})$$

We use Paulin's measure monad to represent distributions:

$$\mathcal{D}(A) \quad \stackrel{\text{def}}{=} \quad (A \to [0,1]) \to [0,1]$$

For instance

$$[\![x \stackrel{\$}{\leftarrow} \{\text{true}, \text{false}\}]\!]\ m = \lambda f \cdot \left(\frac{1}{2} f(m[x/\text{true}]) + \frac{1}{2} f(m[x/\text{false}])\right)$$

To compute probabilities, just measure the characteristic function of the event:

$$\Pr\left[c, m : A\right] \stackrel{\text{def}}{=} [\![c]\!]\ m\ \mathbb{1}_A$$

What does it take to trust a proof in CertiCrypt

- You need to
  - trust the type checker of Coq
  - trust the definition of the language semantics
  - make sure the security statement and the computational assumption (a few lines in Coq) are what you expect it to be
- You don't need to
  - understand or even read the proof
  - trust proof tactics, program transformations
  - trust program logics, wp-calculus
  - be an expert in Coq

Our proof in CertiCrypt I

Game CPA :
$L_1, L_2, L_3 \leftarrow \text{nil};$
$P \xleftarrow{\$} \mathbb{G}_1^+;\ a \xleftarrow{\$} \mathbb{Z}_q^+;$
$P_{pub} \leftarrow aP;$
$(m_0, m_1, ID_{\mathcal{A}}) \leftarrow \mathcal{A}_1(P, P_{pub});$
$d \xleftarrow{\$} \{0,1\};$
$y \leftarrow \mathcal{E}(ID_{\mathcal{A}}, m_d);$
$d_{\mathcal{A}} \leftarrow \mathcal{A}_2(y)$

Oracle $\mathcal{EX}(ID)$:
if $ID \notin L_3$ then
  $L_3 \leftarrow ID :: L_3$
$Q \leftarrow \mathcal{H}_1(ID);$
return aQ

Oracle $\mathcal{H}_1(ID)$:
if $ID \notin \text{dom}(L_1)$ then
  $R \xleftarrow{\$} \mathbb{G}_1^+;$
  $L_1(id) \leftarrow R$
return $L_1(ID)$

Oracle $\mathcal{H}_2(r)$:
if $r \notin \text{dom}(L_2)$ then
  $m \xleftarrow{\$} \{0,1\}^n;$
  $L_2(r) \leftarrow m$
return $L_2(r)$

Game BDH :
$P \xleftarrow{\$} \mathbb{G}_1^+;\ a, b, c \xleftarrow{\$} \mathbb{Z}_q^+;$
$z \leftarrow \mathcal{B}(P, aP, bP, cP)$

$\mathcal{B}(P_0, P_1, P_2, P_3)$:
$L_1, L_2, L_3, V, T \leftarrow \text{nil};$
while $|T| < q_{\mathcal{H}_1}$ do
  $t \xleftarrow{\$} \text{true} \oplus_p \text{false};\ T \leftarrow t :: T$
$P \leftarrow P_0;\ P_{pub} \leftarrow P_1;\ P' \leftarrow P_2;$
$(m_0, m_1, ID_{\mathcal{A}}) \leftarrow \mathcal{A}_1(P, P_{pub});$
$Q_{\mathcal{A}} \leftarrow \mathcal{H}_1(ID_{\mathcal{A}});\ v' \leftarrow V(ID_{\mathcal{A}})^{-1};$
$R \xleftarrow{\$} \{0,1\}^n;\ y \leftarrow (v'P_3, R);$
$d_{\mathcal{A}} \leftarrow \mathcal{A}_2(y);$
$i \xleftarrow{\$} [1..|L_2|];$ return $\text{fst}(L_2[i])$

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if $ID \notin \text{dom}(L_1)$ then
  $v \xleftarrow{\$} \mathbb{Z}_q^+;$
  $V(ID) \leftarrow v;$
  if $T[|L_1|]$ then
    $L_1(ID) \leftarrow vP'$
  else
    $L_1(ID) \leftarrow vP$
return $L_1(ID)$

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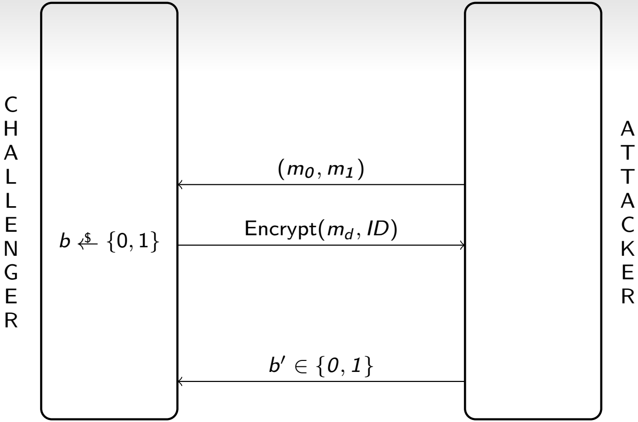
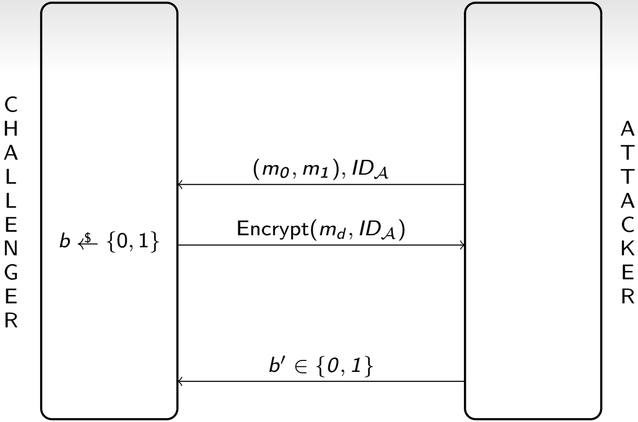
Semantic security of an **IBE** scheme



An **IBE** scheme is *IND-ID-CPA-secure* iff

$$\forall \mathcal{A} \bullet \mathrm{PPT}(\mathcal{A}) \implies \left| \Pr\left[b = b'\right] - \frac{1}{2} \right| \text{ is negligible}$$

Semantic security of an **IBE** scheme



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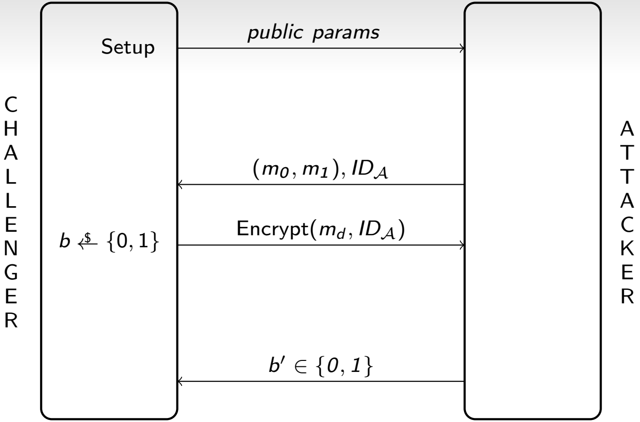

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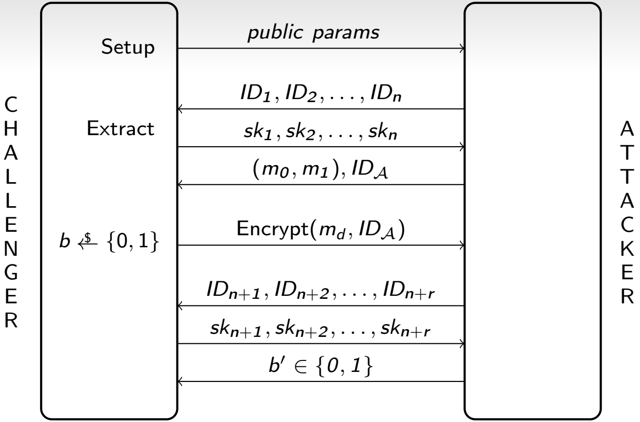

An **IBE** scheme is *IND-ID-CPA-secure* iff

$$\forall \mathcal{A} \bullet \mathrm{PPT}(\mathcal{A}) \wedge \Pr\left[\bigwedge_{i=1}^{m} id_i \neq id_{\mathcal{A}}\right] = 1 \implies \left|\Pr\left[b = b'\right] - \frac{1}{2}\right| \text{ is negligible}$$